The calculation of jerk-free cams for IC engines

Frank Rieg

July 25, 2022

1 Overview

The contour of cams for controlling the valve train of internal combustion engines can be obtained in different ways:

1. Measuring existing, proven cams.

2. Adoption of tables from literature, e.g. [3].

3. Construction of the cam from circular arcs or tangents, e.g. [1] and [3]. Here, however, it is very likely that unwanted leaps that lead to sudden loads will occur during the acceleration process.

4. Calculation and design of jerk-free cams: Here there are only kinks, but no more leaps in the acceleration curve. Various methods have become known here. An older, but very proven and established method goes back to Dipl.-Ing. Dietrich Kurz and Dipl.-Ing. Wolf-Dieter Bensinger, both (at that time) Daimler-Benz AG, Germany, which will be briefly described here so that the background of the two programs CAM_KUBE (for Kurz/Bensinger) and CAM_HAPE for MS Windows can be understood. The use of these two programs by the author is described from page 5 onwards.

2 Theory of calculating jerk-free cams according to Kurz/Bensinger

The valve lift curve is defined from 0° to 90° and is symmetrical from 91° to 180°. The peak of the valve lift curve is at 90°. The curve is now divided into several sections within the range 0°-90°:

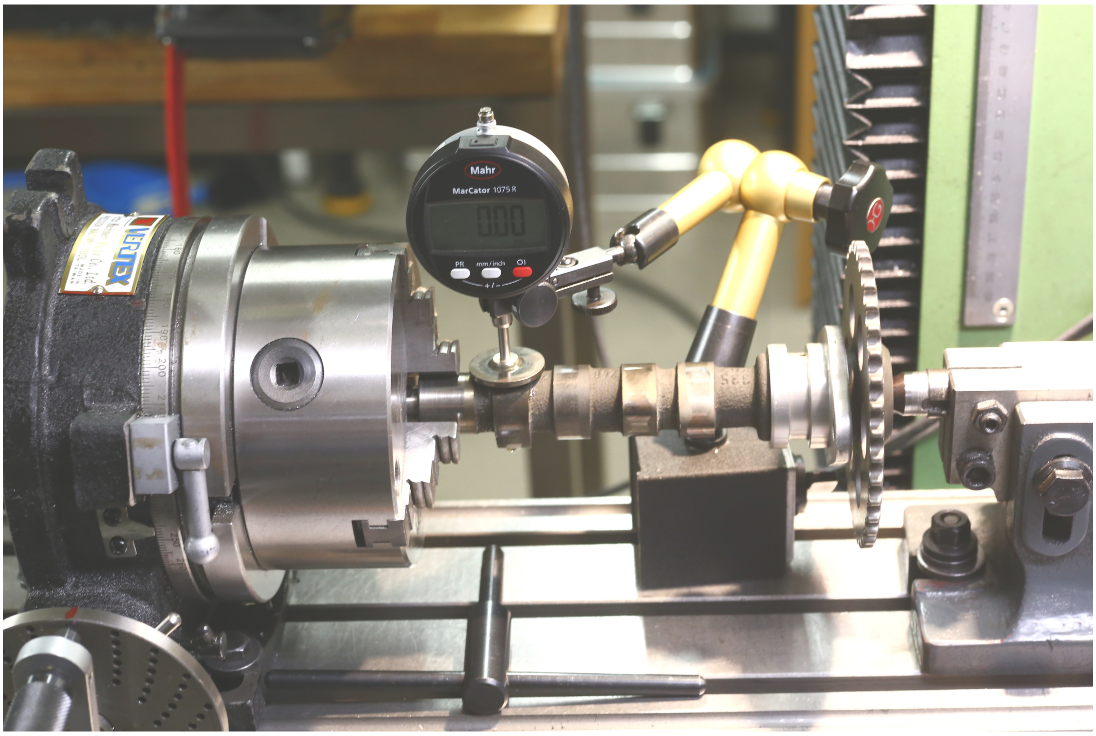
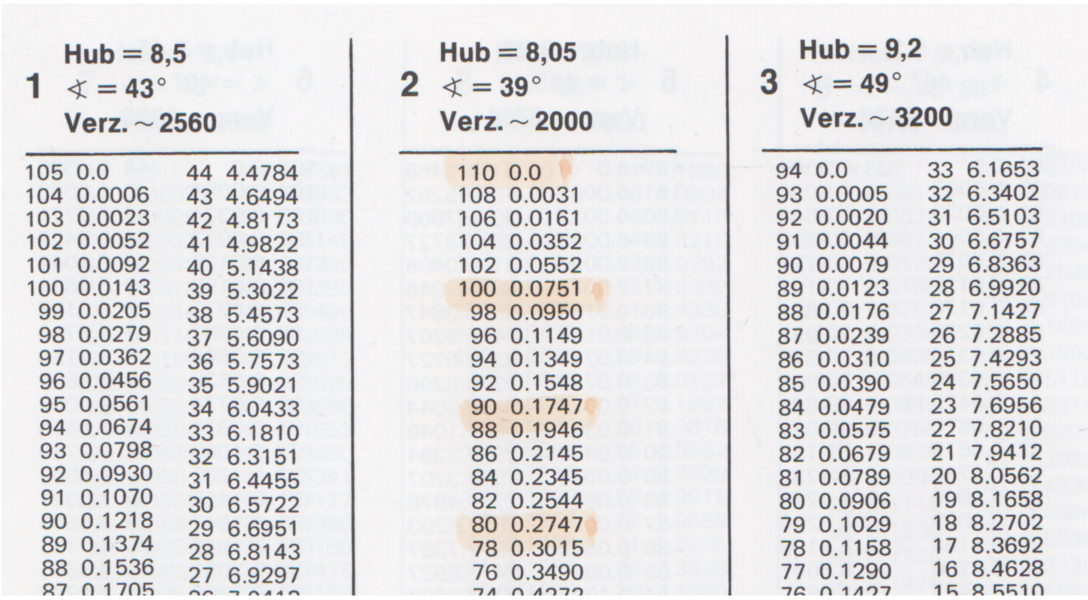


Figure 1: Measuring the cam of a BMW camshaft.



1 Hub = 8,5 ∢ = 43° Verz. ∼ 2560

105	0.0	44	4.4784
104	0.0006	43	4.6494
103	0.0023	42	4.8173
102	0.0052	41	4.9822
101	0.0092	40	5.1438
100	0.0143	39	5.3022
99	0.0205	38	5.4573
98	0.0279	37	5.6090
97	0.0362	36	5.7573
96	0.0456	35	5.9021
95	0.0561	34	6.0433
94	0.0674	33	6.1810
93	0.0798	32	6.3151
92	0.0930	31	6.4455
91	0.1070	30	6.5722
90	0.1218	29	6.6951
89	0.1374	28	6.8143
88	0.1536	27	6.9297
87	0.1705	26	[illegible]

2 Hub = 8,05 ∢ = 39° Verz. ∼ 2000

110	0.0
108	0.0031
106	0.0161
104	0.0352
102	0.0552
100	0.0751
98	0.0950
96	0.1149
94	0.1349
92	0.1548
90	0.1747
88	0.1946
86	0.2145
84	0.2345
82	0.2544
80	0.2747
78	0.3015
76	0.3490
74	0.4272

3 Hub = 9,2 ∢ = 49° Verz. ∼ 3200

94	0.0	33	6.1653
93	0.0005	32	6.3402
92	0.0020	31	6.5103
91	0.0044	30	6.6757
90	0.0079	29	6.8363
89	0.0123	28	6.9920
88	0.0176	27	7.1427
87	0.0239	26	7.2885
86	0.0310	25	7.4293
85	0.0390	24	7.5650
84	0.0479	23	7.6956
83	0.0575	22	7.8210
82	0.0679	21	7.9412
81	0.0789	20	8.0562
80	0.0906	19	8.1658
79	0.1029	18	8.2702
78	0.1158	17	8.3692
77	0.1290	16	8.4628
76	0.1427	15	8.5510

Figure 2: Excerpt from tables of the well-known book by Ludwig Apfelbeck, 13th edition, pg.205 [3].

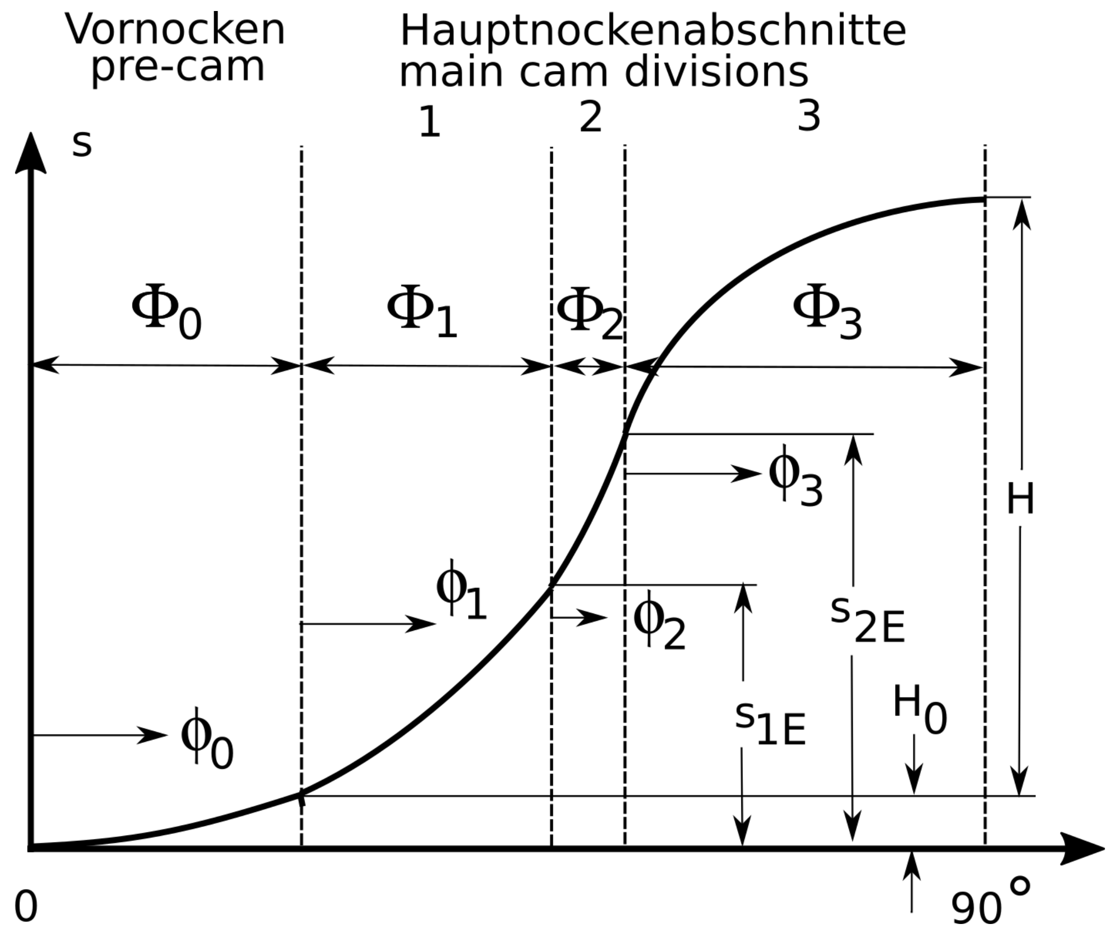


Figure 3: Path curve for a jerk-free cam according to [1] or [2].

0. section, the so-called *pre-cam area*. It is used to bridge the valve clearance and to bring cams and pickups into clean, force-fitting contact: from 0° to an angle Φ_0 with the current angle ϕ_0 the valve lift curve represented by a quarter period sine wave:

$$s_0 = H_0(1 - cos\frac{\pi}{2\Phi_0}\phi_0) \tag{1}$$

The first derivation gives the plot of the velocity from 0° to an angle Φ_0:

$$s_0' = \frac{v_0}{\omega} = H_0\frac{\pi}{2\Phi_0}sin\frac{\pi}{2\Phi_0}\phi_0 \tag{2}$$

The second derivation gives the acceleration curve from 0° to an angle Φ_0:

$$s''_0 = \frac{v_0}{\omega^2} = H_0(\frac{\pi}{2\Phi_0})^2 cos\frac{\pi}{2\Phi_0}\phi_0 \tag{3}$$

1st section, the so-called *1st main cam area*: from Φ_0 to an angle Φ_1 with the current angle $\phi_{\ 1}$ the valve lift curve is represented by a slanted half-period sine curve:

$$s_1 = H_0 + c_{11}\phi_1 - c_{12} sin\frac{\pi}{\Phi_1}\phi_1 \tag{4}$$

$$s'_1 = \frac{v_1}{\omega} = c_{11} - c_{12}\frac{\pi}{\Phi_1} cos\frac{\pi}{\Phi_1}\phi_1 \tag{5}$$

$$s''_1 = \frac{a_1}{\omega^2} = c_{12}(\frac{\pi}{\Phi_1})^2 sin\frac{\pi}{\Phi_1}\phi_1 \tag{6}$$

2nd Section, the so-called *2nd main cam area*: from $\Phi_0 + \Phi_1$ to an angle Φ_2 with the current angle ϕ_2 the valve lift curve is represented by an oblique sine curve of a quarter period:

$$s_2 = s_{1E} + c_{21}\phi_2 + c_{22} sin\frac{\pi}{2\Phi_2}\phi_2 \tag{7}$$

$$s'_2 = \frac{v_2}{\omega} = c_{21} + c_{22}\frac{\pi}{2\Phi_2} cos\frac{\pi}{2\Phi_2}\phi_2 \tag{8}$$

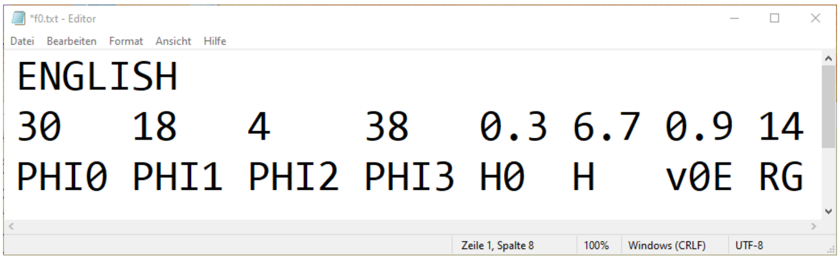
$$s''_2 = \frac{a_2}{\omega^2} = -c_{22}(\frac{\pi}{2\Phi_2})^2 sin\frac{\pi}{2\Phi_2}\phi_2 \tag{9}$$

3rd section, the so-called *3rd main cam area*: from $\Phi_0 + \Phi_1 + \Phi_2$ to an angle Φ_3 with the current angle ϕ_3 the valve lift curve is mapped by a 4th degree polynomial:

$$s_3 = s_{2E} + c_{31}(\Phi_3 - \phi_3)^4 - c_{32}(\Phi_3 - \phi_3)^2 + c_{33} \tag{10}$$

$$s'_3 = \frac{v_3}{\omega} = -4c_{31}(\Phi_3 - \phi_3)^3 + 2c_{32}(\Phi_3 - \phi_3) \tag{11}$$

$$s''_3 = \frac{a_3}{\omega^2} = 12c_{31}(\Phi_3 - \phi_3)^2 - 2c_{32} \tag{12}$$


*f0.txt - Editor
Datei Bearbeiten Format Ansicht Hilfe

```
ENGLISH
30   18   4    38   0.3 6.7 0.9 14
PHI0 PHI1 PHI2 PHI3 H0  H   v0E RG
```

Zeile 1, Spalte 8 100% Windows (CRLF) UTF-8

Figure 4: Input file f0.txt for the example from [1], pg.44.

Now the constants c are calculated (more upon this in [1]), and the following applies: At the end of one cam section and at the beginning of the next cam section, distance, speed and acceleration must be the same. Furthermore, at the end of the pre-cam section, a speed s'_{0E} is specified, and the following must apply: $s_{3E} = H_0 + H$. The ratio $\frac{\Phi_2}{\Phi_3}$ should be chosen to be 0.1 to 0.15 according to [1].

3 Programs CAM_KUBE and CAM_SHAPE for calculating jerk-free cams according to Kurz/Bensinger

The example from [1] starting with p.44 is taken as a basis:

$\Phi_0 = 30°, \Phi_1 = 18°, \Phi_2 = 4°, \Phi_3 = 38°$

$H_0 = 0.3, H = 6.7, s'_{0E} = 0.9$

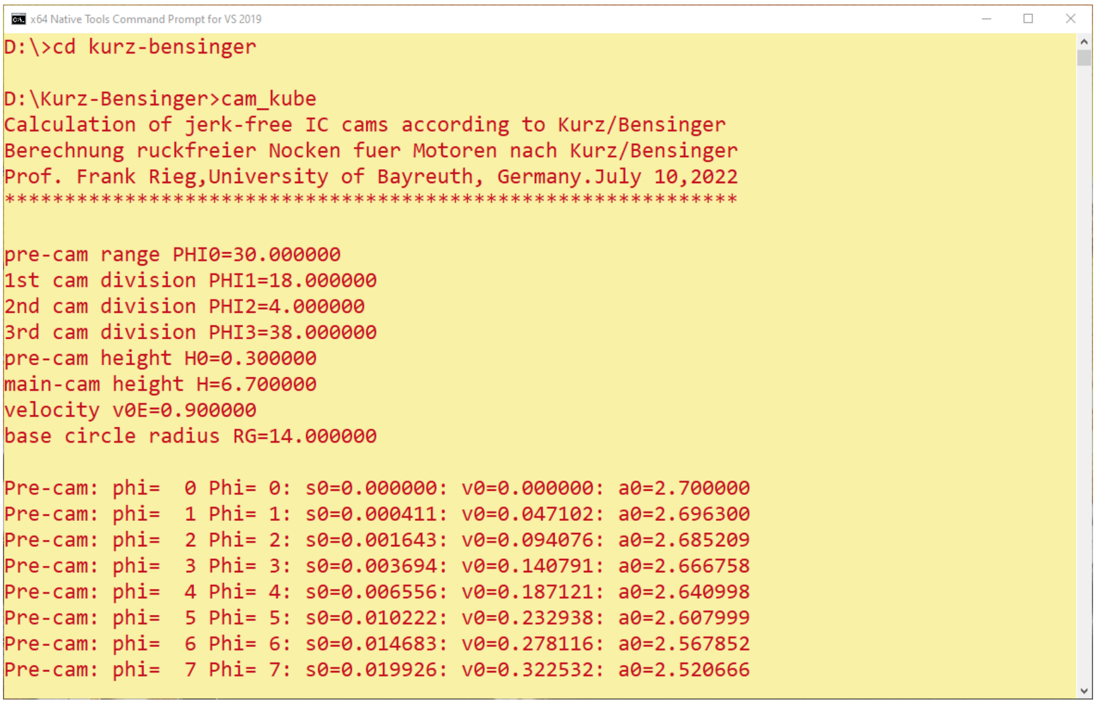
In addition, we assume: base circle radius $R_G = 14$

The input file for CAM_KUBE and CAM_SHAPE is *f0.txt*, fig. 4. It consists of three lines, whereby line 3 is not read; it only serves as input help. For this example, it looks like fig. 4. The language in the first line is selected with the keywords GERMAN or ENGLISH. In line 2, $\Phi_0, \Phi_1, \Phi_2, \Phi_3$,$H_0, H, s'_{0E} = v_{0E}$ and R_G are replaced in this order, each with at least one space separated, entered.

Open a Windows command prompt and start *CAM_KUBE*, see fig. 5.

You will see various outputs on the screen, however, the two curvature values are particularly important and must not be negative under any circumstances, see fig. 6.

The output files *f1.txt* and *f2.txt* are created. *f1.txt*, see fig. 7, contains angles, displacements and the *ω-related* values for speed and acceleration. You may import



```
x64 Native Tools Command Prompt for VS 2019
D:\>cd kurz-bensinger

D:\Kurz-Bensinger>cam_kube
Calculation of jerk-free IC cams according to Kurz/Bensinger
Berechnung ruckfreier Nocken fuer Motoren nach Kurz/Bensinger
Prof. Frank Rieg,University of Bayreuth, Germany.July 10,2022
*************************************************************

pre-cam range PHI0=30.000000
1st cam division PHI1=18.000000
2nd cam division PHI2=4.000000
3rd cam division PHI3=38.000000
pre-cam height H0=0.300000
main-cam height H=6.700000
velocity v0E=0.900000
base circle radius RG=14.000000

Pre-cam: phi=  0 Phi= 0: s0=0.000000: v0=0.000000: a0=2.700000
Pre-cam: phi=  1 Phi= 1: s0=0.000411: v0=0.047102: a0=2.696300
Pre-cam: phi=  2 Phi= 2: s0=0.001643: v0=0.094076: a0=2.685209
Pre-cam: phi=  3 Phi= 3: s0=0.003694: v0=0.140791: a0=2.666758
Pre-cam: phi=  4 Phi= 4: s0=0.006556: v0=0.187121: a0=2.640998
Pre-cam: phi=  5 Phi= 5: s0=0.010222: v0=0.232938: a0=2.607999
Pre-cam: phi=  6 Phi= 6: s0=0.014683: v0=0.278116: a0=2.567852
Pre-cam: phi=  7 Phi= 7: s0=0.019926: v0=0.322532: a0=2.520666
```

Figure 5: The screen output of *CAM_KUBE*.

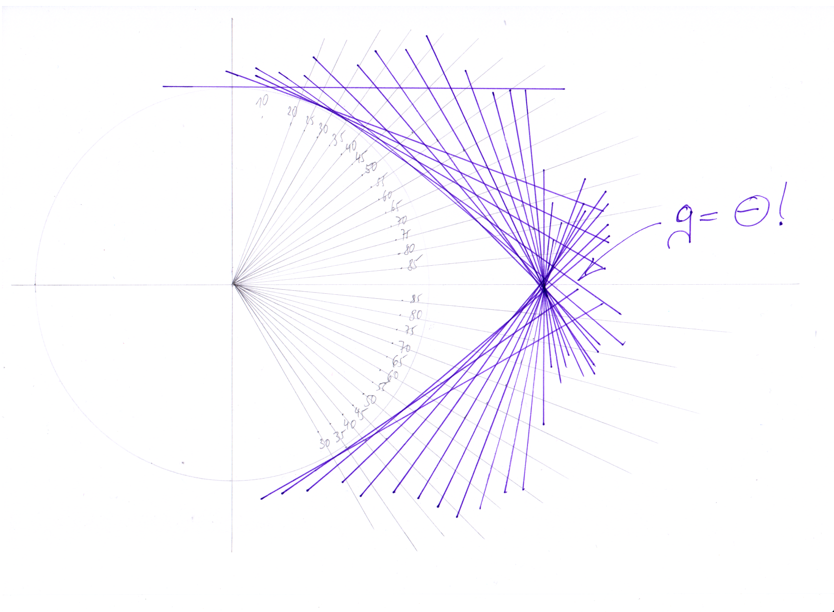


Figure 6: The radius of curvature of the cam tip becomes negative!

phi	displacement	velocity/omega	acc./omega**2
0	+0.0000000E+00	+0.0000000E+00	+2.7000000E+00
1	+4.1114150E-04	+4.7102471E-02	+2.6962997E+00
2	+1.6434391E-03	+9.4075836E-02	+2.6852090E+00
3	+3.6935151E-03	+1.4079135E-01	+2.6667584E+00
4	+6.5557503E-03	+1.8712095E-01	+2.6409982E+00
5	+1.0222300E-02	+2.3293767E-01	+2.6079993E+00
6	+1.4683113E-02	+2.7811592E-01	+2.5678520E+00
7	+1.9925964E-02	+3.2253187E-01	+2.5206663E+00
8	+2.5936482E-02	+3.6606378E-01	+2.4665717E+00
9	+3.2698193E-02	+4.0859233E-01	+2.4057163E+00
10	+4.0192563E-02	+4.5000095E-01	+2.3382669E+00
11	+4.8399050E-02	+4.9017615E-01	+2.2644086E+00
12	+5.7295161E-02	+5.2900780E-01	+2.1843436E+00
13	+6.6856512E-02	+5.6638947E-01	+2.0982914E+00
14	+7.7056896E-02	+6.0221869E-01	+2.0064879E+00
15	+8.7868355E-02	+6.3639727E-01	+1.9091848E+00
16	+9.9261255E-02	+6.6883152E-01	+1.8066487E+00

Figure 7: Output file f1.txt with angles, displacements and related speeds and accelerations.

270+phi	90-phi	milling depth
270	90	0.000000
272	88	0.001643
274	86	0.006556
276	84	0.014683
278	82	0.025936
280	80	0.040193
282	78	0.057295
284	76	0.077057
286	74	0.099261

Figure 8: Output file f2.txt: Milling data for tangential milling.

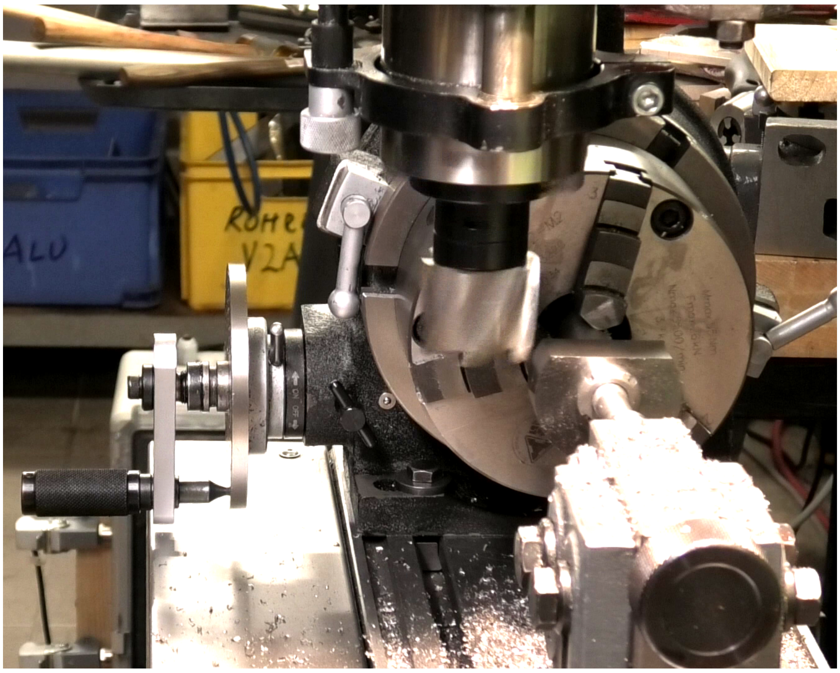


Figure 9: Tangential milling of the cam on a universal milling machine with a commercially available dividing head.

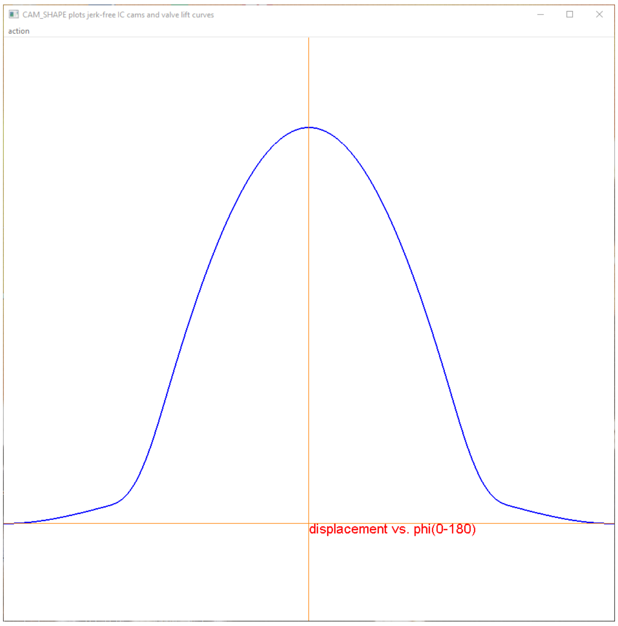


Figure 10: Displacement curve of the example cam taken from [1].

Figure 11: Velocity curve for the example cam taken from [1].

Figure 12: Acceleration curve for the example cam taken from [1].

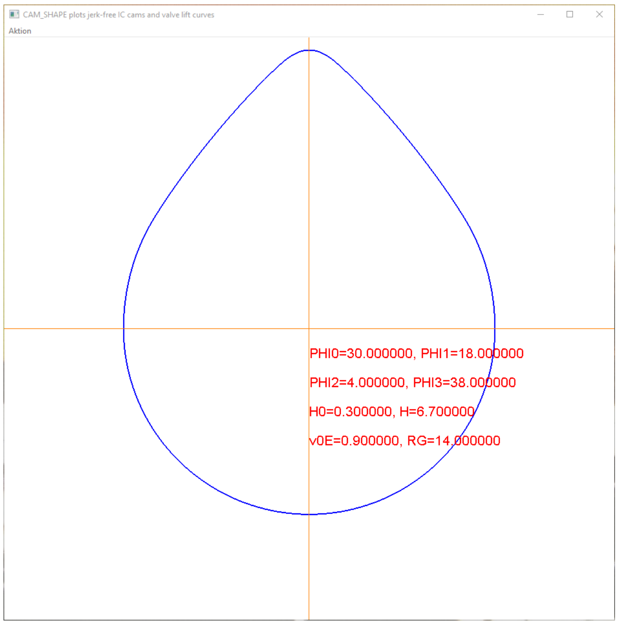


Figure 13: Shape of the example cam taken from [1].

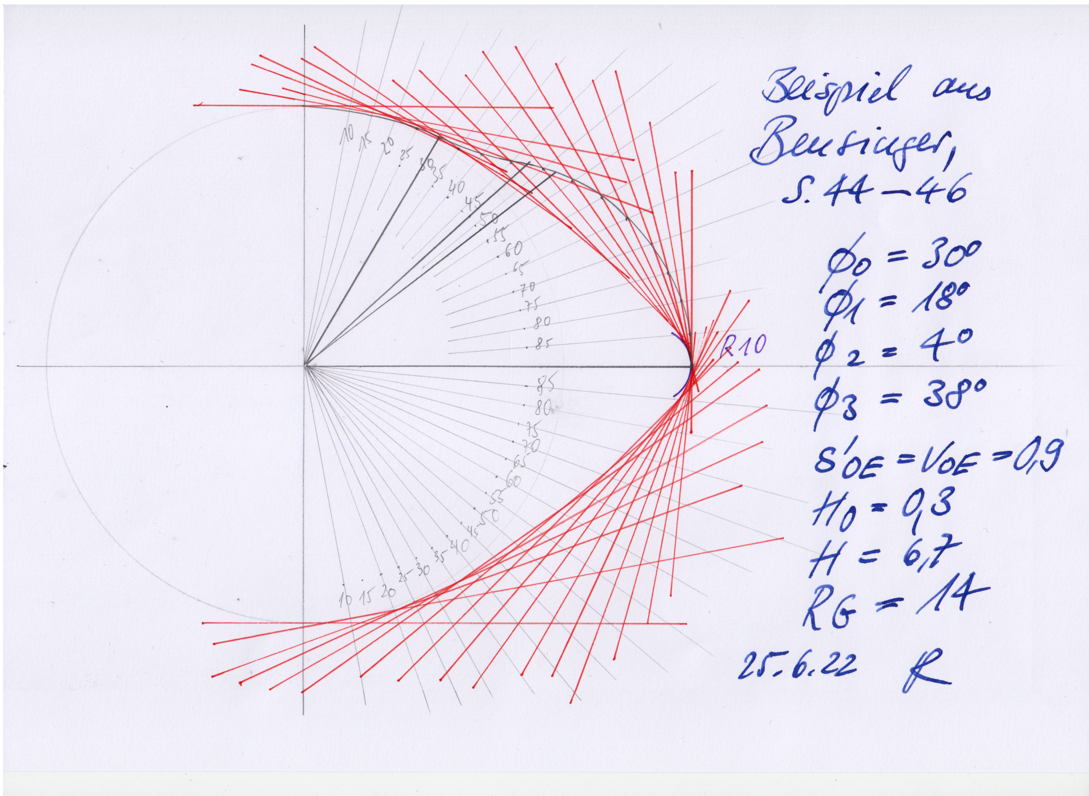


Figure 14: Shape of the example cam taken from [1], drawn by hand.

easily this file into e.g. EXCEL and display the curves there if necessary. The output file *f2.txt* directly contains the milling data in 2° steps for tangential milling on a universal milling machine, see fig. 8 and fig. 9. The tip of the cam is defined as 0° and you will start milling at 180°, i.e. exactly opposite the tip of the cam. You move up to 270° on the cam base circle with the radius R_G, i.e. without height feed. Beginning from 272° you will move the milling head upwards, respectively. Then harden and temper the milled cam and smooth it with emery cloth - that's all it takes. But you could also grind the cam on a home-made cam grinding machine: you can find videos 209, 211 and 220 on our YouTube channel LSCAD.

After *CAM_KUBE* has run, you can use the program *CAM_SHAPE* to plot the actual cam shape for **flat cam followers** such as bucket tappets. Completely different cam shapes result for roller tappets with the same valve lift curves! Roller tappets are not yet provided in *CAM_SHAPE* and *CAM_KUBE*.

The displacement, speed and acceleration curves of the example cam from [1], related to cam coordinates, output by *CAM_SHAPE* can be seen in fig. 10, fig. 11 and fig. 12; the actual cam shape in fig. 13. You can also draw the cam shape by hand without *CAM_SHAPE* by plotting the displacements starting from the base circle and drawing a straight line perpendicular to it, i.e. tangents. The group of these tangents form the envelope of the cam, see fig. 14.

References

[1] Bensinger, W.-D.: Die Steuerung des Gaswechsels in schnellaufenden Verbrennungsmotoren. 2.Auflage. Springer Verlag. Berlin, Heidelberg: 1968

[2] Urlaub, A.:Verbrennungsmotoren. 2.Auflage. Springer Verlag. Berlin, Heidelberg: 1994

[3] Apfelbeck, L.: Wege zum Hochleistungs-Viertaktmotor. 13.Auflage. Motorbuch-Verlag. Stuttgart: 1997.